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Sincere voting in an electorate with heterogeneous preferences[☆]

Boris Ginzburg

Department of Economics, Universidad Carlos III de Madrid, Calle Madrid 126, 28903 Getafe, Spain

H I G H L I G H T S

- Voters need to choose between a reform and the status quo.
- There is a binary state of the world, and voters receive private signals about it.
- Some voters prefer the reform in the first state, others – in the second.
- When fractionalisation is sufficiently high, an equilibrium exists in which almost all voters vote sincerely.

A B S T R A C T

Much of the theoretical literature on voting with private information finds that voters do not vote sincerely at the equilibrium. Yet there is little empirical support for this result. This paper shows that when the electorate is sufficiently divided, sincere voting is an equilibrium strategy for an arbitrarily large proportion of voters.

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1. Introduction

When imperfectly informed voters go to polls, an intuitive conjecture might be that they vote sincerely – select the alternative that they would like to win, given their information. [Austen-Smith and Banks \(1996\)](#) and [Feddersen and Pesendorfer \(1997\)](#) demonstrated, however, that sincere voting is not in general a rational strategy. Yet there is little evidence that voters in real-life elections act contrary to their private signals. In fact, existing empirical evidence suggests that voters do vote sincerely ([Degan and Merlo, 2009](#)).

This paper provides game-theoretic foundations for the sincere voting hypothesis. In the paper, voters choose between a reform and the status quo. There is a binary state of the world and two

types of voters. Each voter receives an imperfect continuous signal about the state. A voter wants the reform to be chosen if and only if the state corresponds to his type. Hence, the electorate is fractionalised – voters of the two types have conflicting preferences. This is different from the standard setup,¹ in which changing the state moves voters' preferred alternative in the same direction.^{2,3}

The paper shows that as fractionalisation becomes sufficiently large, the expected share of voters who vote against their private signals goes to zero. The result holds for any size of the electorate, any threshold voting rule, any signal structure,⁴ and any prior

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E-mail address: boris.ginzburg@uc3m.es.

URL: <http://sites.google.com/site/borgin/>.

¹ See e.g. [Koriyama and Szentes \(2009\)](#), [Goertz and Maniquet \(2014\)](#), [Ellis \(2016\)](#).

² In [Goertz and Maniquet \(2011\)](#), [Bouton and Castanheira \(2012\)](#); [Bouton et al. \(2016\)](#), and [Ferrari \(2016\)](#), some voters are partisans and always prefer one of the alternatives – but preferences of voters who care about the state are still monotone in the state.

³ [Kim and Fey \(2007\)](#) and [Bhattacharya \(2013\)](#) examine a setting with adversarial preferences, but they study large elections and do not focus on sincere voting behaviour.

⁴ Subject to some technical assumptions.

belief. Hence, even in very asymmetric settings, sincere voting is an equilibrium if the electorate is sufficiently divided.

Conflicting preferences are present in many situations. For instance, a trade reform can increase wages in some sectors while decreasing them in others. A voter will support the reform in a referendum if and only if wages in his sector will rise. Similarly, in an election there may be uncertainty about the challenger's preferred policies relative to those of the incumbent. Left-wing voters will prefer the challenger if she is more left-wing than the incumbent, while right-wing voters will have the opposite preferences.

Several other papers look at situations in which sincere voting is an equilibrium strategy. In Krishna and Morgan (2012), sincere voting can happen when voting is voluntary. In Damiano et al. (2015), sincere voting is driven by the possibility of a recount. In Acharya and Meiwitz (2016), some voters are uninformed, and when their number can be large, informed voters vote sincerely. Ferrari (2016) studies sincere voting in a setting with public signals and partisan voters. In contrast, this paper focuses not on voting procedure or information structure, but on conflicting preferences as a factor that induces sincere voting.

2. Model

An electorate of $n + 1$ voters vote on a reform. Each voter i has a type $x_i \in \{0, 1\}$, which represents her preferences. There is an unknown state of the world $\omega \in \{0, 1\}$; let $\Pr(\omega = 1) = p$. If the reform is approved, voter i receives a payoff $u(x_i, \omega)$. Let $u(x_i, \omega) = 1$ if $x_i = \omega$; and $u(x_i, \omega) = -1$ if $x_i \neq \omega$. Thus, each voter gains from the reform if the state corresponds to her type, and loses otherwise.⁵ If the reform is rejected, each voter receives a payoff of zero.

In the beginning, nature selects the state. Then nature draws voter types; each type is drawn independently, and the probability that a voter's type is 1 is $\gamma \in (0, 1)$. We can think of γ as a measure of fractionalisation: the closer γ is to 0.5, the more fractionalised the electorate is.⁶

Each voter knows γ , but does not know the realised types, other than her own.⁷ After observing her type, each voter i receives an independent private signal s_i . Signals are distributed continuously with full support on some set $C \subseteq \mathbb{R}$. Let F_ω and f_ω be, respectively, the cdf and the density of s_i conditional on the state being $\omega \in \{0, 1\}$. I will make two assumptions about the signal distributions:

1. Monotone likelihood ratio: $h(s_i) \equiv \frac{f_0(s_i)}{f_1(s_i)}$ is strictly increasing in s_i .
2. Non-triviality: there exist signals s, s' such that $h(s) < \frac{p}{1-p} < h(s')$.

The first assumption says that the inverse of the likelihood ratio exists and that the state is more likely to be zero when signal realisation is high. The second assumption says that the signal is potentially informative: there exist signal realisations under which

⁵ Thus, no voter favours or opposes the reform in both states. Note that if such voters were present, they would always vote the same way at an equilibrium. Hence, introducing them has the same effect as varying the voting rule.

⁶ For a society consisting of different groups, the index of fractionalisation represents the probability that two randomly drawn individuals belong to different groups. When there are two groups, fractionalisation is higher when the groups are more similar in size. See Montalvo and Reynal-Querol (2005).

⁷ We can think of γ as representing a distribution of voters' types reported by a noisy opinion poll. Alternatively, if n is small – for example, when the decision is made by a committee – we can think of γ as reflecting a distribution of types within a population, which is known (for example, from polls). Then $n + 1$ members of that population are selected into the committee. If n is large, γ may correspond to a distribution of types reported by a noisy opinion poll. On the other hand, if $n \rightarrow \infty$, γ simply becomes a known distribution of voters' preferences.

the posterior belief that the state is zero is above 0.5, and also signal realisations under which the posterior belief that the state is zero is below 0.5.

After observing her signal, each voter votes for or against the reform. The reform is adopted if the number of votes in favour of it is strictly larger than some number k . Thus, $k = \frac{n}{2}$ represents simple majority rule, while $k = n$ represents unanimity rule. After the vote, payoffs are realised depending on whether the reform is adopted and on the state.

Denote by $a_i(x_i, s_i) \in [0, 1]$ voter i 's action (the probability of voting for the reform) at the equilibrium as a function of her signal and her type. As usual in voting games, this game has trivial equilibria in which no voter is ever pivotal. Hence, the analysis will focus on equilibria in which each voter is pivotal with a positive probability and which are symmetric in the sense that $a_i(x_i, s_i) = a(x_i, s_i)$, $\forall i$ (that is, in which two voters with the same type and the same signal select the same action).⁸ I will call these *symmetric pivotal equilibria*.

3. Results

3.1. Sincere voting

A sincere voter only considers her signal when making the decision. Hence, she backs the reform if and only if

$$E[u(x_i, \omega) | s_i] \geq 0.$$

Given a signal s_i , we have $\Pr(\omega = 0 | s_i) = \frac{(1-p)f_0(s_i)}{(1-p)f_0(s_i) + pf_1(s_i)}$. Hence, if $x_i = 0$, voter i backs the reform whenever $(1-p)f_0(s_i) - pf_1(s_i) \geq 0$, i.e. whenever $h(s_i) \geq \frac{p}{1-p}$. Similarly, if $x_i = 1$, i backs the reform whenever $pf_1(s_i) - (1-p)f_0(s_i) \geq 0$, i.e. whenever $h(s_i) \leq \frac{p}{1-p}$.

3.2. Equilibrium voting

Suppose that each voter behaves strategically. Then she knows that her vote can only affect the outcome if she is pivotal – i.e. if exactly k other voters have voted in favour of the reform. Denote this event by *piv*. A voter i who is strategic will vote to support the reform if and only if

$$E[u(x_i, \omega) | s_i, \text{piv}] \geq 0.$$

At equilibrium, let q_ω denote the probability that a randomly selected voter votes for the reform when the true state is ω . Therefore, when $\omega = 0$, each voter is pivotal with probability $\binom{n}{k} q_0^k (1 - q_0)^{n-k}$; and when $\omega = 1$, a voter is pivotal with probability $\binom{n}{k} q_1^k (1 - q_1)^{n-k}$. Then the following holds:

Lemma 1. *In any symmetric pivotal equilibrium, q_0 and q_1 are distinct from 0 and 1.*

Proof. Suppose that the opposite holds. Because signal distributions have full support, $q_\omega = 0$ or $q_\omega = 1$ means that a voter votes the same way upon receiving any signal. Then $q_1 = q_0$. If $q_0 = q_1 = 1$, the probability that a voter is pivotal is either 0 (if $k \neq n$), or 1 (if $k = n$). Similarly, if $q_0 = q_1 = 0$, the probability that a voter is pivotal is either 0 (if $k \neq 0$), or 1 (if $k = 0$). If the probability that a voter is pivotal is 0, this is not a symmetric pivotal equilibrium. If that probability is 1, the best response of each voter is to vote sincerely – but then non-triviality ensures that his vote varies with signal, so q_0 and q_1 cannot equal 0 or 1. Thus, $q_\omega = 0$ or $q_\omega = 1$ leads to a contradiction.

⁸ It will be shown that an equilibrium with these properties exists.

Having established this, we can describe voters' equilibrium strategy with the following lemma, the proof of which is in the [Appendix](#):

Lemma 2. *In every symmetric pivotal equilibrium, the strategy of every voter i satisfies the following:*

- $a_i(0, s_i) = 1$ if and only if $h(s_i) \geq M$
- $a_i(1, s_i) = 1$ if and only if $h(s_i) \leq M$

where

$$M \equiv \frac{p}{1-p} \frac{q_1^k(1-q_1)^{n-k}}{q_0^k(1-q_0)^{n-k}}.$$

Recall that [Lemma 1](#) ensures that $q_0, q_1 \in (0, 1)$ – hence, M exists and is distinct from zero. Note that M is a function of q_0 and q_1 , which are endogenous to the equilibrium. At a symmetric pivotal equilibrium, a voter backs the reform if she is of type 0 and receives a signal above $h^{-1}(M)$, or if she is of type 1 and receives a signal below $h^{-1}(M)$.⁹ Thus, $q_0 = (1-\gamma)(1-F_0[h^{-1}(M)]) + \gamma F_0[h^{-1}(M)]$, and $q_1 = (1-\gamma)(1-F_1[h^{-1}(M)]) + \gamma F_1[h^{-1}(M)]$. Rearranging and substituting into the expression for M yields the following equality:

$$M = \frac{p}{1-p} \left(\frac{1-\gamma - (1-2\gamma)F_1[h^{-1}(M)]}{1-\gamma - (1-2\gamma)F_0[h^{-1}(M)]} \right)^k \times \left(\frac{\gamma + (1-2\gamma)F_1[h^{-1}(M)]}{\gamma + (1-2\gamma)F_0[h^{-1}(M)]} \right)^{n-k}. \quad (1)$$

This equality must hold at any symmetric pivotal equilibrium. Furthermore, for any value of M for which (1) holds, no player wants to deviate from strategies described in [Lemma 2](#). Thus, if (1) holds for some value of M , then that value determines a symmetric pivotal equilibrium. The following lemma, the proof of which is in the [Appendix](#), proves that such a value of M exists:

Lemma 3. *Equality (1) holds for at least one value of M .*

This implies the existence of a symmetric pivotal equilibrium.

3.3. Sincere voting at the equilibrium

Consider a symmetric pivotal equilibrium. From the results in [Section 3.1](#) and [Lemma 2](#) it follows that if $s_i \geq h^{-1}\left(\frac{p}{1-p}\right)$ and $s_i \geq h^{-1}(M)$, then at the equilibrium a strategic voter i is voting sincerely, backing the reform if $x_i = 0$ and opposing the reform if $x_i = 1$. If $s_i \leq h^{-1}\left(\frac{p}{1-p}\right)$ and $s_i \leq h^{-1}(M)$, voter i also votes sincerely, supporting the reform if $x_i = 1$ and opposing it if $x_i = 0$. A voter is not voting sincerely at the equilibrium if and only if her signal falls between $h^{-1}\left(\frac{p}{1-p}\right)$ and $h^{-1}(M)$. Let μ_ω denote the probability that a randomly chosen voter is not voting sincerely when the true state is $\omega \in \{0, 1\}$. Then,

$$\mu_\omega = \left| F_\omega[h^{-1}(M)] - F_\omega\left[h^{-1}\left(\frac{p}{1-p}\right)\right] \right|.$$

The following proposition then establishes the main result of the paper:

Proposition. *At any symmetric pivotal equilibrium, $\lim_{\gamma \rightarrow \frac{1}{2}} \mu_\omega = 0$ for any $\omega \in \{0, 1\}$ and any p, n, k, F_0, F_1 .*

⁹ In the subsequent proof of [Lemma 3](#) it will be shown that $h^{-1}(M)$ is well-defined at equilibrium.

Proof. If $\gamma \rightarrow \frac{1}{2}$, then for any M the right-hand side of (1) approaches $\frac{p}{1-p}$. Thus, at the limit, $M = \frac{p}{1-p}$, and $\mu_\omega = 0$.

In words, at a symmetric pivotal equilibrium, the expected fraction of voters not voting sincerely becomes vanishingly small as the electorate becomes sufficiently fractionalised.

4. Conclusions

The paper has examined the behaviour of imperfectly informed voters. Much of the previous literature concludes that in such settings, voters will act against their private information. This paper shows that when voters have conflicting preferences, sincere voting can be an equilibrium. In particular, when the electorate is sufficiently divided, an arbitrarily large fraction of voters is voting sincerely. This result holds for any prior, any size of the electorate, any voting rule, and any pair of signal distributions that satisfies the assumptions in [Section 2](#).

Appendix

A.1. Proof of [Lemma 2](#)

By Bayes' law, we have

$$\begin{aligned} \Pr(\omega = 0 \mid s_i, \text{piv}) &= \frac{(1-p)f_0(s_i) \binom{n}{k} q_0^k(1-q_0)^{n-k}}{(1-p)f_0(s_i) \binom{n}{k} q_0^k(1-q_0)^{n-k} + pf_1(s_i) \binom{n}{k} q_1^k(1-q_1)^{n-k}} \end{aligned}$$

or

$$\Pr(\omega = 0 \mid s_i, \text{piv}) = \frac{h(s_i)}{h(s_i) + M}.$$

Then we have:

$$E[u(0, \omega) \mid s_i, \text{piv}] = \frac{h(s_i) - M}{h(s_i) + M}$$

$$E[u(1, \omega) \mid s_i, \text{piv}] = \frac{M - h(s_i)}{h(s_i) + M}.$$

The conditions under which these are greater than zero are equivalent to the conditions in [Lemma 2](#).

A.2. Proof of [Lemma 3](#)

Denote the right-hand side of (1) by $B(M)$. It is defined when $h^{-1}(M)$ exists – that is, when $M \in (\inf[h(s)], \sup[h(s)])$. When $M \rightarrow \inf[h(s)] = h(\inf[C])$, we have $h^{-1}(M) \rightarrow \inf(C)$. Then $F_0[h^{-1}(M)]$ and $F_1[h^{-1}(M)]$ both go to 0, hence $B(M) \rightarrow \frac{p}{1-p}$. Similarly, when $M \rightarrow \sup[h(s)] = h(\sup[C])$, we have $h^{-1}(M) \rightarrow \sup(C)$. Then $F_0[h^{-1}(M)]$ and $F_1[h^{-1}(M)]$ both go to 1, hence $B(M) \rightarrow \frac{p}{1-p}$. Thus, when $M \in (\inf[h(s)], \sup[h(s)])$, the function $M - B(M)$ takes values close to $\inf[h(s)] - \frac{p}{1-p}$, and also values close to $\sup[h(s)] - \frac{p}{1-p}$. Non-triviality assumption implies that $\inf[h(s)] < \frac{p}{1-p} < \sup[h(s)]$. Thus, the function $M - B(M)$ takes both negative and positive values on the $(\inf[h(s)], \sup[h(s)])$ interval, and since this function is continuous, it should by Bolzano's theorem also take a value of zero on this interval. Therefore, there exists at least one value of M at which $M = B(M)$, i.e. at which (1) holds.

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